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Isospin breaking in the yield of heavy meson pairs in e^+e^- annihilation near threshold

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Abstract

We revisit the problem of interplay between the strong and the Coulomb interaction in the charged-to-neutral yield ratio for $B\bar{B}$ and $D\bar{D}$ pairs near their respective thresholds in e^+e^- annihilation. We consider here a realistic situation with a resonant interaction in the isospin $I = 0$ channel and a nonresonant strong scattering amplitude in the $I = 1$ state. We find that the yield ratio has a smooth behavior depending on the scattering phase in the $I = 1$ channel. The same approach is also applicable to the $K\bar{K}$ production at the $\phi(1020)$ resonance, where the Coulomb effect in the charged-to-neutral yield ratio is generally sensitive to the scattering phases in both the isoscalar and the isovector channels. Furthermore, we apply the same approach to the treatment of the effect of the isotopic mass difference between the charged and neutral mesons and argue that the strong-scattering effects generally result in a modification to the pure kinematical effect of this mass difference.

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1 Introduction

The $J^{PC} = 1^{--}$ resonances near the new flavor thresholds: $\Upsilon(4S)$, $\psi(3770)$, and $\phi(1020)$ are the well known sources in e^+e^- experiments of pairs of the new-flavor mesons: respectively $B\bar{B}$, $D\bar{D}$, and $K\bar{K}$. A number of experimental approaches depends on the knowledge of the relative yield of pairs of charged and neutral mesons:

$$R^{c/n} = \frac{\sigma(e^+e^- \to P^+P^-)}{\sigma(e^+e^- \to P^0\bar{P}^0)} \; , \tag{1}$$

where P stands for the pseudoscalar meson, i.e. B, D, or K, and dedicated measurements of such ratio have been done at the $\Upsilon(4S)$ resonance [1] at $\psi(3770)$ [2] and at $\phi(1020)$ [3].

The values of the ratio $R^{c/n}$ at all three discussed resonances are close to one due to these resonances being isotopic scalars, and it is the deviation of the discussed ratio from one that presents phenomenological interest. This deviation is generally contributed by the following factors: the isospin violation due to the Coulomb interaction between the charged mesons and due to the isotopic mass difference between charged and neutral mesons, and, in the case of the $K\bar{K}$ production at the $\phi(1020)$ resonance, a non-negligible nonresonant isovector production amplitude. The latter effect can be studied and described as the "tail of the ρ resonance", while the isospin breaking due to the mass difference is usually accounted for as a kinematical effect in the P wave production cross section factor p^3, where p is the the c.m. momentum of each of the mesons. The Coulomb effect has attracted a considerable theoretical attention. The expression for this effect in the ratio $R^{c/n}$ in the limit, where the resonance and the charged mesons are considered as point-like particles [4] has the simple textbook form:

$$\delta R^{c/n} = \frac{\pi\alpha}{2v}, \tag{2}$$

with α being the QED constant and v the velocity of each of the (charged) mesons in the c.m. frame. However for the production of the real-life mesons the analysis is complicated by the charge form factors of the mesons[5], by the form factor in the vertex of interaction of the resonance with the meson pair [5, 6] and generally by the strong interaction between the mesons [7, 8, 9]. In particular, it has been argued [8, 9] that the modification of the Coulomb effect by the strong (resonant) interaction between the mesons is quite significant. The previously considered picture of the strong interaction was however somewhat unrealistic. Namely, it has been assumed [8, 9] that the wave function in the $I = 1$ state of the meson pair is vanishing at short but finite distances, which would correspond to a singular behavior

of the strong interaction at finite distances. In this paper we derive the formulas for the Coulomb effect in the ratio $R^{c/n}$ under the standard assumption about the strong scattering amplitude in the channels with $I = 0$ and $I = 1$. We find that in the case of the $\Upsilon(4S)$ and $\psi(3770)$ resonances, where the heavy meson pairs are produced by the isotopically singlet electromagnetic current of the corresponding heavy quark, the strong-interaction effect in the Coulomb correction depends on the scattering phase δ_1 in the $I = 1$ channel and is a smooth function of the energy across the resonance, while in the case of the Kaon production at and near the $\phi(1020)$ there is also a smooth dependence on the nonresonant part of the strong scattering phase δ_0 in the isoscalar channel inasmuch as there is a contribution of the isovector production amplitude at these energies. In either case we find that the behavior of the Coulomb effect is smooth on the scale of the resonance width, unlike the behavior previously found [8, 9] under less realistic assumptions.

We further notice that essentially the same calculation can be applied to considering the effect on the ratio $R^{c/n}$ of the isotopic mass difference Δm between the charged and neutral mesons, at least in the first order in Δm, by considering the mass difference as a perturbation by a (constant) potential. In this way we find that the result coincides with the linear in Δm term in the ratio of the kinematical factors p^3 only in the limit of vanishing strong scattering phase. Once the latter phase is taken into account, there arises a correction whose relative contribution is determined by the parameter $(p\,a)$ with a being the characteristic range of the strong interaction. We therefore conclude that the conventionally used p^3 approximation for this effect may be somewhat applicable to the $K\bar{K}$ production at the $\phi(1020)$ resonance, where $p \approx 120\,\text{MeV}$, but becomes quite questionable for the $D\bar{D}$ production at the $\psi(3770)$, where $p \approx 280\,\text{MeV}$.

The strong-scattering phase in the P-wave state of mesons produced in e^+e^- annihilation near the threshold is proportional to p^3. We therefore expect the discussed effects of the strong interaction in the ratio $R^{c/n}$ to exhibit a measurable variation with energy. A measurement of this variation can thus provide an information on the strong scattering phases, which is not readily available by other means.

The material in the paper is organized as follows. In Sec. 2 we consider the production of meson pairs by an isosinglet source and derive the formula for the correction to $R^{c/n}$ due to a generic isospin-violating interaction potential $V(r)$ viewed as a perturbation. In Sec. 3 we generalize this treatment to the situation where the source is a coherent mixture of $I = 0$ and $I = 1$. The specific expressions corresponding to the Coulomb interaction and the isotopic

mass difference are considered in Sec. 4. Sec. 5 contains phenomenological estimates of the constraints on the parameters of the strong interaction between heavy mesons based on the currently available data [1, 2] for $B\bar{B}$ and $D\bar{D}$ production. Finally, in Sec. 6 we summarize our results.

2   General formulas for an isoscalar source

We start with considering the behavior of the scattering wave functions of a meson-antimeson pair in the limit of exact isotopic symmetry, i.e. neglecting any Coulomb effects and the isotopic mass difference. We adopt the standard picture (see e.g. in the textbook [10]), where the strong interaction is confined within the range of distances $r < a$, so that beyond that range, at $r > a$ the motion of the mesons is free. The two relevant independent solutions to the Schrödinger equation at $r > a$ for the radial wave function in the P wave are the free outgoing wave

$$f(pr) = \left(1 + \frac{i}{pr}\right) e^{ipr} \tag{3}$$

and its complex-conjugate, $f^*(pr)$, describing the incoming wave. A general wave function of a pair of neutral mesons, $\phi_n(r)$ as well as of a pair of charged mesons, $\phi_c(r)$, in this region is a linear superposition of these two solutions.

In the region of strong interaction, i.e. at $r < a$, the isotopic symmetry selects as independent channels the states with definite isospin, $I = 0$ and $I = 1$, corresponding to the wave functions $\phi_0 = \phi_c + \phi_n$ and $\phi_1 = \phi_c - \phi_n$. The detailed behavior of the $I = 0$ and $I = 1$ wave functions inside the strong interaction region is not important for the present treatment, and the important point is that the non-singular at $r = 0$ 'inner' wave functions match at $r = a$ particular linear superpositions of the incoming and outgoing waves (which superpositions in fact correspond to standing waves):

$$\chi_0(r) = e^{i\delta_0} f(pr) + e^{-i\delta_0} f^*(pr) \,,$$
$$\chi_1(r) = e^{i\delta_1} f(pr) + e^{-i\delta_1} f^*(pr) \,, \tag{4}$$

where δ_0 and δ_1 are the strong scattering phases in respectively the isoscalar and isovector states.

Consider now the production of meson pairs by a source localized inside the region of strong interaction, i.e. $r < a$, such as e.g. the electromagnetic current. The wave function

of the produced meson pairs at $r \leq a$ is then determined by both the source and the strong interaction, and the relevant solution to the Schrödinger equation is chosen by the requirement that asymptotically at large distances, $r \to \infty$, only an outgoing wave is present. Let us first consider the simple case where the relevant electromagnetic current is a pure isotopic singlet, which is the case for $D\bar{D}$ and $B\bar{B}$ pair production. Then in the limit of exact isotopic symmetry the outgoing waves for the 'n' and the 'c' channels have exactly the same amplitude, which for our present purpose can be chosen as one:

$$\phi_c^{(0)}(r) = f(pr) \ \text{ and } \ \phi_n^{(0)}(r) = f(pr) \ \text{ at } r \to \infty \ , \tag{5}$$

where the superscript (0) stands for the approximation of exact isotopic symmetry. It can be noted that the approximation of the free motion beyond the region of the strong interaction in fact makes the expressions in Eq.(5) applicable at all $r > a$, i.e. all the way down to the matching point $r = a$. It is helpful to notice for a later discussion that at the matching point the $I = 1$ wave function is vanishing while the $I = 0$ function $\phi_0^{(0)}$ contains only the outgoing wave. When continued into the strong interaction region, i.e. at $r < a$, the function ϕ_0 evolves into the solution determined by the strong interaction and the source.

The isospin-violating effects of the Coulomb interaction and of the mass difference Δm between the charged and neutral mesons can be generally described as being due to a presence of an extra potential $V(r)$ in the 'c' channel beyond the region of the strong interaction: $V = -\alpha/r$ for the Coulomb interaction effect and a constant potential $V = 2\,\Delta m$ describing the mass difference. In other words the wave function ϕ_n of the 'n' channel is still determined at $r > a$ by the radial Schrödinger equation for free P-wave motion[2], while the equation for the 'c' channel function ϕ_c reads as

$$\left(\frac{\partial^2}{\partial r^2} + p^2 - m\,V(r) - \frac{2}{r^2} \right) \phi_c(r) = 0 \ . \tag{6}$$

It is assumed throughout the present consideration that the isospin-breaking potential exists only at distances beyond the range of the strong interaction, i.e. that $V(r)$ has support only at $r > a$. The justification for such treatment is that in the region of the strong force

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[2]Clearly, in the considered here first order in the isospin violation only the difference of the interaction between the two channels is important, thus any such difference can be relegated to one channel, while keeping the other one unperturbed. Also, any effect of the mass difference in the kinetic term p^2/m is of order v^2/c^2 as compared to the discussed here effect of Δm in the overal energy difference between the two channels, and is totally neglected in our treatment.

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small isospin-violating effects are compared to the energy of the strong interaction, so that the contribution of any such effects arising at $r < a$ is very small, while in the region $r > a$ the relative contribution of the potential $V(r)$ is determined by its ratio to the kinetic energy of the mesons, which is small near the threshold.

It should be emphasized that although the interaction at distances $r > a$ is present only in the 'c' channel, the wave functions in *both* channels are modified in comparison with those in Eq.(5), as a result of the coupling between channels imposed by the boundary conditions at $r = a$. According to the setting of the problem of production of the meson pairs by a localized source, the appropriate modified functions are those containing at $r \to \infty$ only the outgoing waves

$$\phi_c \to (1 + x)\, f(pr), \quad \phi_n \to (1 + y)\, f(pr) \,, \tag{7}$$

where the (complex) coefficients x and y arise due to the potential V, and are proportional to V in the considered here first order of perturbation theory. These coefficients determine the ratio of the production amplitudes: $A_c/A_n = 1 + x - y$, and the discussed here modification of the yield ratio:

$$R^{c/n} = 1 + 2\,\mathrm{Re}\,x - 2\,\mathrm{Re}\,y \,. \tag{8}$$

The modified wave function in both channels is subject to two conditions:
i: The channel with neutral mesons has only an outgoing wave at all $r > a$. In other words, the expression for $\phi_n(r)$ in Eq.(7) is valid at all r down to $r = a$;
ii: The wave function of the channel with isospin $I = 1$ at $r \le a$ should be proportional to the standing-wave solution matching the function χ_1 in Eq.(4), since there is no source for the $I = 1$ state of the meson pairs.
These two conditions are sufficient to fully determine the modified functions at $r > a$ and thus to find the coefficients x and y.

The first order in $V(r)$ perturbation of the wave function in the channel with charged mesons is found in the standard way, using the P wave Green's function $G_+(r, r')$ satisfying the equation

$$\left(\frac{\partial^2}{\partial r^2} + p^2 - \frac{2}{r^2} \right) G_+(r, r') = \delta(r - r') \,, \tag{9}$$

and the condition that $G_+(r, r')$ contains only an outgoing wave when either of its arguments goes to infinity. The Green's function is constructed from two solutions of the homogeneous equation, i.e. from the functions $f(pr)$ and $f^*(pr)$, as

$$G_+(r, r') = \frac{1}{2\,i\,p}\, [f(pr)\, f^*(pr')\, \theta(r - r') + f(pr')\, f^*(pr)\, \theta(r' - r)] \,, \tag{10}$$

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where θ is the standard unit step function. The perturbation $\delta\phi_c$ is then found as

$$\delta\phi_c(r) = m \int_a^\infty G_+(r,r') \, V(r') \, f(pr') \, dr' . \tag{11}$$

One readily finds from this explicit form of the solution that $\delta\phi_c$ contains only the outgoing wave at asymptotic distances $r \to \infty$:

$$\delta\phi_c \,|_{r\to\infty} = -\frac{i}{2v} \, f(pr) \int_a^\infty V(r') \, |f(pr')|^2 \, dr' , \tag{12}$$

so that the coefficient x is purely imaginary:

$$x = -\frac{i}{2v} \int_a^\infty V(r') \, |f(pr')|^2 \, dr' \tag{13}$$

and gives no contribution to the ratio of the production rates $R^{c/n}$ described by Eq.(8)[3].

Consider now the matching of the wave functions at $r = a$. In this region of r one has $r < r'$ in the integral in Eq.(11) so that the correction in the 'c' channel has only an incoming wave:

$$\delta\phi_c(r) \,|_{r\to a} = \eta \, f^*(pr) \tag{14}$$

with

$$\eta = -\frac{i}{2v} \int_a^\infty V(r') \, [f(pr')]^2 \, dr' . \tag{15}$$

The wave functions $\phi_0 = \phi_c + \phi_n$ and $\phi_1 = \phi_c - \phi_n$ corresponding to the states with isospin $I = 0$ and $I = 1$ are then found as

$$\phi_0 \,|_{r\to a} = 2f(pr) + y \, f(pr) + \eta \, f^*(pr) \quad \text{and} \quad \phi_1 \,|_{r\to a} = \eta \, f^*(pr) - y \, f(pr) . \tag{16}$$

One can now apply the condition **ii** to determine the coefficient y. Indeed, the condition for the wave function ϕ_1 at $r \to a$ to be proportional to $f^*(pr) + e^{2i\delta_1} f(pr)$ requires y to be given by

$$y = -\eta \, e^{2i\delta_1} . \tag{17}$$

Upon substitution in Eq.(8) this yields

$$R^{c/n} = 1 + \frac{1}{v} \, \text{Im} \left[e^{2i\delta_1} \int_a^\infty e^{2ipr} \left(1 + \frac{i}{pr} \right)^2 V(r) \, dr \right] . \tag{18}$$

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[3]It can be noticed that the integral in Eq.(13) is divergent, which corresponds to the infrared-divergent behavior of the perturbation for the phase of the wave function, logarithmic for the Coulomb interaction and linear for a constant potential. This slight technical difficulty can be readily resolved, for our present purposes, by introducing an infrared regularizing factor $\exp(-\lambda r)$ in the potential and setting $\lambda \to 0$ in the end result.

3 Mixed isoscalar and isovector source

The formula (18) gives the general expression for the isospin-breaking effect in the considered yield ratio for the case where the mesons are produced by an isoscalar source. The presented consideration can also be extended to a situation where the source is a general coherent mixture of an isoscalar and isovector. The specific isotopic composition of the source determines the ratio of the coefficients of the amplitudes of the running outgoing waves in the $I = 1$ and $I = 0$ channels at the matching point $r = a$, which ratio we denote as A_1/A_0, thus defining A_1 and A_0 as the production amplitudes in the respective channels (in the limit of exact isotopic symmetry). In this situation the generalization of the expressions in Eq.(5) for radial wave functions in the 'outer' region $r > a$ in the zeroth order in the isospin violation can be written as

$$\phi_c^{(0)}(r) = (A_0 + A_1) f(pr) \quad \text{and} \quad \phi_n^{(0)}(r) = (A_0 - A_1) f(pr) \ . \tag{19}$$

The isospin violation in the asymptotic form of these wave functions at $r \to \infty$ can then be parametrized, similarly to Eq.(7), by complex coefficients x and y as

$$\phi_c \to (A_0 + A_1)(1 + x) f(pr), \qquad \phi_n \to (A_0 - A_1)(1 + y) f(pr) \ , \tag{20}$$

so that the yield ratio is found from

$$R^{c/n} = \left| \frac{A_0 + A_1}{A_0 - A_1} \right|^2 (1 + 2\operatorname{Re} x - 2\operatorname{Re} y \ ) \ . \tag{21}$$

The coefficient x, similarly to the previous discussion and the equation (13), is purely imaginary and in fact does not contribute in Eq.(21), while the coefficient y is found from the appropriately modified conditions on the wave functions. Namely, the previously discussed condition **i** remains applicable, so that the asymptotic expression in Eq.(20) for the 'n' channel function remains valid in the entire 'outer' region $r > a$ down to the matching point $r = a$. In order to allow for the isovector component of the source the condition **ii** has to be modified as will be described few lines below.

The perturbation by the potential $V(r)$ of the 'c' channel wave function at the matching point $r = a$ is readily found, similarly to Eq.(14), as

$$\delta\phi_c(r) \mid_{r \to a} = \eta (A_0 + A_1) f^*(pr) \tag{22}$$

with η given by Eq.(15).

One can now write the expressions for the resulting 'outer' wave functions in the isotopic channels at the matching point:

$$\phi_0(r) \mid_{r \to a} = 2\,A_0\,f(pr) + \eta\,(A_0 + A_1)\,f^*(pr) + y\,(A_0 - A_1)f(pr) =$$
$$\left[2\,A_0 + y\,(A_0 - A_1) - \eta\,(A_0 + A_1)\,e^{2i\delta_0}\right]f(pr) + \eta\,(A_0 + A_1)\,e^{i\delta_0}\,\chi_0(r) \qquad (23)$$

and

$$\phi_1(r) \mid_{r \to a} = 2\,A_1\,f(pr) + \eta\,(A_0 + A_1)\,f^*(pr) - y\,(A_0 - A_1)f(pr) =$$
$$\left[2\,A_1 - y\,(A_0 - A_1) - \eta\,(A_0 + A_1)\,e^{2i\delta_1}\right]f(pr) + \eta\,(A_0 + A_1)\,e^{i\delta_1}\,\chi_1(r) , \qquad (24)$$

with χ_0 and χ_1 being the standing wave functions from Eq.(4) in the corresponding isotopic channels, which when evolved in the region of strong interaction contain no singularity at $r = 0$. The remaining parts in the latter expressions for the functions ϕ_0 and ϕ_1 describe the proper *running* outgoing waves. These parts, when continued down in r into the strong interaction region evolve to match the source at $r < a$. The ratio of the amplitudes of the isovector and the isoscalar running waves is determined by the isotopic composition of the source, and by the isotopically symmetric propagation through the strong-interaction region. Thus the ratio of the amplitudes of these waves at $r = a$ does not depend on the isospin-breaking effects at $r > a$ and should be equal to A_1/A_0. Applying this condition to the isotopic wave functions given by the expressions (23) and (24), one finds the equation for the coefficient y:

$$\frac{2\,A_1 - y\,(A_0 - A_1) - \eta\,(A_0 + A_1)\,e^{2i\delta_1}}{2\,A_0 + y\,(A_0 - A_1) - \eta\,(A_0 + A_1)\,e^{2i\delta_0}} = \frac{A_1}{A_0} \; . \qquad (25)$$

This equation in fact replaces in this more general situation the previously discussed condition **ii**, which condition and the ensuing result in Eq.(17) are readily recovered in the limit $A_1/A_0 = 0$ from Eq.(25).

Considering that both y and η are of the first order in the potential V, it is sufficient to use the linear expansion of the equation (25) in y and η, finding in this way the solution for y in the form

$$y = -\eta \frac{A_0\,e^{2i\delta_1} - A_1\,e^{2i\delta_0}}{A_0 - A_1} \; , \qquad (26)$$

and thus arriving at the final formula for the relative yield:

$$R^{c/n} = \left|\frac{A_0 + A_1}{A_0 - A_1}\right|^2 \left\{1 + \frac{1}{v}\,\mathrm{Im}\left[\frac{A_0\,e^{2i\delta_1} - A_1\,e^{2i\delta_0}}{A_0 - A_1}\int_a^\infty e^{2ipr}\left(1 + \frac{i}{pr}\right)^2 V(r)\,dr\right]\right\} . \qquad (27)$$

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Given that $A_0 = |A_0|\, e^{i\delta_0}$ and $A_1 = |A_1|\, e^{i\delta_1}$, the amplitude-dependent factor in this formula can also be written in terms of the real ratio $\rho = |A_1/A_0|$ as

$$\frac{A_0\, e^{2i\delta_1} - A_1\, e^{2i\delta_0}}{A_0 - A_1} = e^{2i\delta_1}\, \frac{1 - \rho\, e^{i(\delta_0 - \delta_1)}}{1 - \rho\, e^{-i(\delta_0 - \delta_1)}} \ . \tag{28}$$

4   The Coulomb and the mass-difference effects

The general formulas in Eq.(18) and (27) can now be applied to a discussion of the specific isospin-breaking effects in the e^+e^- production of meson pairs at and near the threshold resonances. We start with considering the effect of the Coulomb interaction. In a detailed treatment of this correction one should include the realistic form factors of the mesons, which cut off at short distances the difference in the electromagnetic interactions between the charged and neutral mesons. In the present discussion we replace for simplicity the gradual cutoff of the Coulomb interaction by an abrupt cutoff at an effective range $r = a_c$, where generally $a_c \geq a$ [4]. The master integral with the Coulomb potential $V(r) = -\alpha/r$ in the equations (18) and (27) then takes the form

$$\int_{a_c}^{\infty} e^{2ipr}\left(1 + \frac{i}{pr}\right)^2 V(r)\, dr =$$
$$\alpha\left\{\left[\frac{\cos 2pa_c}{2(pa_c)^2} + \frac{\sin 2pa_c}{pa_c} - Ci(2pa_c)\right] + i\left[\frac{\pi}{2} - \frac{\cos 2pa_c}{pa_c} + \frac{\sin 2pa_c}{2(pa_c)^2} - Si(2pa_c)\right]\right\} =$$
$$\alpha\left\{\left[\frac{1}{2\,(pa_c)^2} - \ln(2\,pa_c) + 1 - \gamma_E\right] + i\left[\frac{\pi}{2} + \frac{pa_c}{3}\right] + O\left[(pa_c)^2\right]\right\} \ , \tag{29}$$

where the integral sine and cosine are defined in the standard way:

$$Si(z) = \int_0^z \sin t\, \frac{dt}{t} \quad \text{and} \quad Ci(z) = -\int_z^{\infty} \cos t\, \frac{dt}{t} \ ,$$

and $\gamma_E = 0.577\ldots$ is the Euler's constant. The latter line in Eq.(29) shows few first terms of the expansion of the integral in the parameter (pa_c). This expansion illustrates the behavior of the correction toward the threshold. For the purpose of this illustration one can consider first the simpler expression in Eq.(18). The imaginary part, which determines the discussed Coulomb effect in $R^{c/n}$ in the limit where there is no strong scattering, $\delta_1 \to 0$, is not singular

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[4]As previously mentioned, any extension of the isospin-breaking potential inside the strong interaction region can result only in very small corrections.

at $pa_c \to 0$, and the textbook formula (2) is recovered in this limit. The real part of the integral in Eq.(29) is singular at small pa_c, but it multiplies in Eq.(18) the factor $\sin \delta_1$. The P-wave scattering phase in its turn is proportional at small momenta to p^3: $\delta_1 \sim (pa)^3$, so that the overall contribution of the real part of the integral is not singular at the threshold either. Considering a more general expression for the Coulomb effect for the case of an isotopically mixed source, following from the equation (27), one can readily arrive at the same conclusion that the singular in (pa_c) real part of the integral (29) does not lead to an actual singularity, since it only enters the ratio $R^{c/n}$ multiplied by a combination of the phases δ_0 and δ_1 (cf. Eq.(28)), each vanishing as p^3 toward the threshold.

As previously mentioned, the effect of the isotopic mass difference corresponds to that of a constant potential $V = 2\,\Delta m$ extending from the range of the strong interaction $r = a$ to infinity. The master integral with such potential has the form

$$\int_a^\infty e^{2ipr} \left(1 + \frac{i}{pr}\right)^2 V(r)\, dr =$$
$$-\frac{\Delta m}{p} \left\{ \frac{2 \cos 2\, pa}{pa} + \sin 2\, pa + i \left[\frac{2 \sin 2\, pa}{pa} - \cos 2\, pa \right] \right\} =$$
$$-\frac{\Delta m}{p} \left\{ \frac{2}{pa} - 2\, pa + 3\, i + O\left[(pa)^2 \right] \right\} . \tag{30}$$

In the limit of vanishing strong scattering phases the mass correction to $R^{c/n}$ is determined by only the imaginary part of the integral, which in the limit of small pa thus yields

$$R^{c/n} = 1 - \frac{3\,\Delta m}{v\, p} = 1 - \frac{3\,\Delta m}{E} , \tag{31}$$

where E is the total kinetic energy of the meson pair, and the found expression coincides with the linear in Δm term in the expansion of the usually assumed ratio of the kinematical factors $(p_+/p_0)^3$. Clearly, in the more realistic case of presence of the strong scattering the real part of the integral in Eq.(30) also contributes and the simple kinematical approximation is generally invalidated.

5   Phenomenological estimates

In this section we discuss application of our formulas to interpreting the data on the charged to neutral meson yield ratio $R^{c/n}$ at the near-threshold resonances $\Upsilon(4S)$, $\psi(3770)$ and

$\phi(1020)$. The purpose of this discussion is to illustrate the effect of the strong scatering on the isospin breaking corrections, and we use here the simplified picture of a abrupt cutoff of the Coulomb interaction and of the isotopic mass difference effects. Such simplification generally can be used as long as the parameter (pa) is not large. A detailed analysis should likely involve a model of a gradual cutoff, since the details of the transition become important at lager momenta.

5.1 $\Upsilon(4S)$

The simplest case for the study of the isospin breaking corrections in the relative production of heavy mesons is offered by the $B\bar{B}$ pair production near and at the $\Upsilon(4S)$ resonance. Indeed, this process only is due to the purely isosinglet electromagnetic current of the b quarks, and the isotopic mass difference between the B mesons is very small: $\Delta m_B = -0.33 \pm 0.28 \, \text{MeV}$ [11], so that any deviation of the ratio $R^{c/n}$ from one is essentially entirely due to the Coulomb interaction. On the other hand, the parameter α/v for the Coulomb effect in this case is the largest due to small velocity of the B mesons: at the energy of the $\Upsilon(4S)$ peak $v_B/c \approx 0.06$. In particular, the numerical value in the expression (2) is 0.19. The experimental data [1] however indicate a significantly smaller deviation of $R^{c/n}$ from one. The BaBar data with the smallest errors give $R^{c/n} = 1.006 \pm 0.036 \pm 0.031$. Such behavior is likely a result of a combined effect of the meson and production vertex form factors [5, 6] and of the discussed here modification of the Coulomb correction by the strong scattering phase. These effects can in principle be separated and studied quantitatively by measuring the energy dependence of the ratio $R^{c/n}$ near the $\Upsilon(4S)$ resonance. With the presently available data we can only use a simplified parametrization of the form factor effects by introducing an abrupt cutoff for the Coulomb interaction at $r = a_c \geq a$ and thereby estimate the likely regions in the (a_c, δ_1) plane. Such estimate from the equations (18) and (29) is shown in Fig.1 as a one-sigma area, corresponding to the BaBar data with the statistical and systematic errors added in quadrature: $R^{c/n} = 1.006 \pm .048$. Clearly, more precise data from dedicated measurements of the ratio $R^{c/n}$ are needed for a better understanding of the parameters of strong interaction between the B mesons.
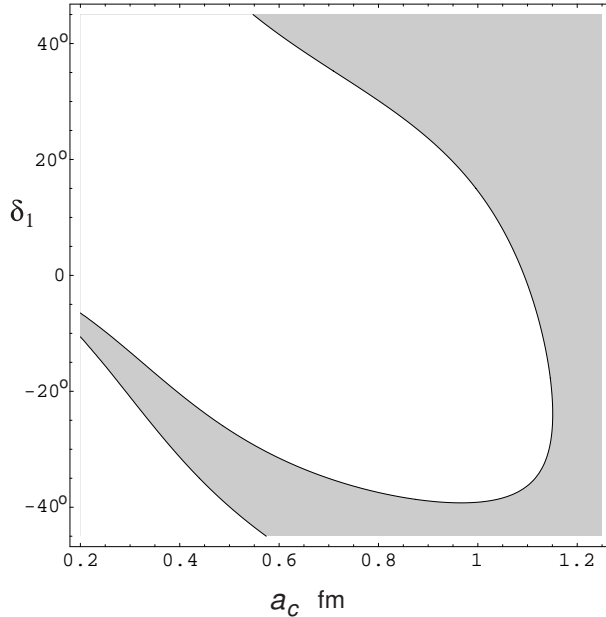
Figure 1: The one sigma area (shaded) in the (a_c, δ_1) plane corresponding to the BaBar data on the $B^+B^-/B^0\bar{B}^0$ yield ratio at the $\Upsilon(4S)$ resonance.

5.2 $\psi(3770)$

The largest isospin-breaking effect in the $D\bar{D}$ production at the $\psi(3770)$ is that due to the mass difference between the charged and the neutral D mesons: $\Delta m_D = 4.78 \pm 0.10\,\text{MeV}$ [11]. The most precise measurements of this process have been done [2] at the energy $\sqrt{s} = 3773\,\text{MeV}$. At this energy the momentum of each charged D meson is $p_+ = 254\,\text{MeV}$ and that for a neutral D meson is $p_0 = 287\,\text{MeV}$. Thus the ratio of the kinematical factors $(p_+/p_0)^3 \approx 0.69$ is significantly less than one. The Coulomb effect is somewhat smaller. Indeed, the velocity of a charged meson at this energy is $v_+/c = 0.135$ and the expression (2) gives numerically 0.085. One can notice that if the kinematical and the Coulomb factors are combined in a straightforward way to estimate $R^{c/n} = (p_+/p_0)^3 \left[1 + \pi\alpha/(2v_+)\right] \approx 0.75$, this would be in a very good agreement with the experimental number [2]: $R^{c/n} = 0.776 \pm 0.024^{+0.014}_{-0.006}$. Thus it is quite likely that at this particular energy there is a considerable cancelation between the strong-interaction effects in the yield ratio, and such cancelation by itself imposes constraints on the parameters of strong interaction between the D mesons, which constraints is interesting to analyze.

An analysis of the strong-interaction effects along the lines discussed in the present paper generally runs into two difficulties. One is that our approach is accurate only in the linear in Δm approximation, while the actual effect of the isotopic mass difference between the D mesons is not very small. However, numerically, the first term in the expansion of the kinematical factor (Eq.(31)) gives 0.67, which is quite close to the mentioned above value 0.69, and it looks like the linear term gives a reasonable approximation. The other point is that the cutoff parameter a_c for the Coulomb interaction at short distances does not necessarily coincide with the range parameter a used for the short-distance cutoff of the effect of the mass difference. However, as previously noted, the Coulomb effect is somewhat small at the energy of the $\psi(3770)$ resonance, and for the purpose of preliminary estimates we set $a_c = a$ in our numerical analysis. In order to allow for possible errors introduced by our approximations in comparing with the data, we linearly add a theoretical uncertainty of 0.03 units to the combined in quadrature statistical and experimental errors. Proceeding in this way we find that the only region in the (a, δ_1) plane at $a < 2\,\text{fm}$ consistent with the CLEO-c data at one sigma level is the one shown in Fig.2.
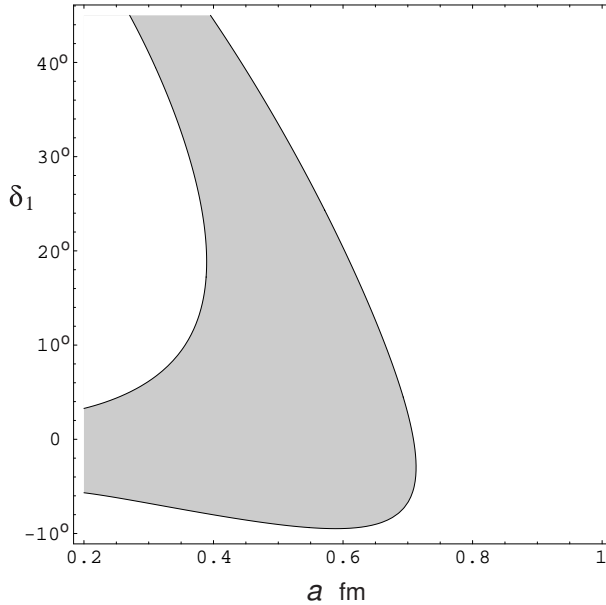

Figure 2: The area (shaded) in the (a, δ_1) plane corresponding to the CLEO-c data on the $D^+D^-/D^0\bar{D}^0$ yield ratio at the $\psi(3770)$ resonance. The uncertainty shown includes a one sigma experimental error with our estimate of the theoretical uncertainty added linearly.

It is interesting to compare the plots in the Figures 1 and 2. In the heavy quark limit applied to both b and c quarks the strong interaction between the heavy mesons should be the same, corresponding to the same range parameters a and a_c. The scattering phase δ_1 for these two systems is generally different due to different masses. However, provided there are no isovector 'molecular' bound states, the sign of the phase should be the same, with the absolute value of the phase for heavier B mesons being larger than for the D mesons. The comparison with the data for the D mesons favors small values of the range parameter, as indicated by Fig.2. If one also assumes that $a_c \approx a$ for the B mesons, the short range of a_c, according to Fig.1, is compatible with the B mesons data at a negative scattering phase δ_1, which sign of δ_1 is also in agreement with the D meson data. A negative sign of δ_1 corresponds to a repulsion, which for the $I = 1$ state of heavy meson pairs can be expected on general grounds [12].

5.3 $\phi(1020)$

We believe that the production of $K\bar{K}$ pairs in e^+e^- annihilation at and near the $\phi(1020)$ resonance merits a separate analysis along the lines discussed in the present paper and using detailed data similar to those in Ref.[3]. As is known, this production receives a small but measurable nonresonant contribution from the isovector part of the electromagnetic current of the u and d quarks, which corresponds to an isotopically mixed source. Furthermore, it has been pointed out [13] that a detailed theoretical analysis of the $K^+K^-/K^0\bar{K}^0$ yield ratio at the $\phi(1020)$ resonance produces a result which possibly is at a meaningful variance with the data.

At present we limit ourselves to noticing that the formula in Eq.(27), applicable in this situation, describes a smooth behavior of the considered isospin breaking effects across the resonance in the $I = 0$ channel. Indeed, the $I = 0$ scattering phase at energy E near the resonance energy E_0 is given by the Breit-Wigner formula

$$e^{2i\delta_0} = \frac{\Delta - i\gamma}{\Delta + i\gamma} e^{2i\tilde{\delta}_0} , \qquad (32)$$

where $\Delta = E - E_0$, $\tilde{\delta}_0$ is the nonresonant scattering phase in the isoscalar channel, and γ is the width parameter. Both $\tilde{\delta}_0$ and γ are smooth functions of the energy proportional to p^3 at small momentum, and $\gamma(E_0)$ determines the resonance width Γ as $\gamma = \Gamma/2$. The ratio of the isovector and isoscalar production amplitudes can then be parametrized near the resonance

as

$$\frac{A_1}{A_0} = \frac{\Delta + i\,\gamma}{\mu}\,e^{i\,(\delta_1 - \tilde{\delta}_0)}\;, \tag{33}$$

where μ is a parameter with dimension of energy: $\mu \sim m_\phi - m_\rho$. The amplitude ratio entering the correction factor in Eq.(27) can then be written in the form

$$\frac{A_0\,e^{2i\delta_1} - A_1\,e^{2i\delta_0}}{A_0 - A_1} = e^{2i\delta_1}\,\frac{\mu - (\Delta - i\,\gamma)\,e^{-i(\delta_1 - \tilde{\delta}_0)}}{\mu - (\Delta + i\,\gamma)\,e^{+i(\delta_1 - \tilde{\delta}_0)}}\;, \tag{34}$$

which manifestly shows that this ratio is a pure phase factor of a complex quantity slowly varying across the $\phi(1020)$ resonance.

6 Summary

We have considered the effects of the isospin breaking by the Coulomb interaction and by the isotopic mass difference in the relative yield $R^{c/n}$ of pairs of charged and neutral mesons near threshold by a compact source, such as in the production of heavy mesons in e^+e^- annihilation. These effects are modified by the strong interaction scattering phases. The general formula for a situation where the source is an arbitrary coherent mixture of an isoscalar and isovector is given by Eq.(27). In particular, for a purely isoscalar source, which is the case for the e^+e^- annihilation into $D\bar{D}$ and $B\bar{B}$ pairs the strong-interaction effect is determined by the scattering phase δ_1 in the $I = 1$ channel (Eq.(18)). As a practical matter we find that under the standard assumptions about the strong scattering amplitudes in the near-threshold resonance region the ratio $R^{c/n}$ has a smooth behavior with energy showing no abnormal rapid variation on the scale of the resonance width. The energy dependence of this ratio is rather determined by the non-resonant scattering scattering phase(s). In the P-wave the phase δ_1 is proportional to p^3, so that a measurement of the behavior ratio $R^{c/n}$ with energy can provide information on this phase, which is not readily accessible by other means. The behavior of the ratio $R^{c/n}$ at larger energies away from the threshold also depends on the details of the onset of the strong interaction between the heavy mesons at short distances and on the behavior of their electromagnetic form factors, and a study of this behavior can provide an insight into these properties of the heavy-light hadrons.

Acknowledgements

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